Lesson 4-2: Triangle Congruence: SSS & SAS

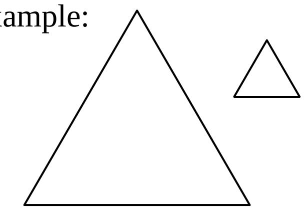
Triangle congruence

Last lesson we started thinking about what it takes to determine if two triangles are congruent. We learned that if all corresponding sides ***and*** all corresponding angles are congruent, then we can say the triangles are congruent.

But do we really need all that? Can we determine congruence if we are missing some pieces?

Is angle congruence good enough?

If I told you that all corresponding angles of two triangles were congruent, but nothing more, could you determine they are congruent? Suppose we have two equiangular triangles (all angles are congruent … and measure 60). Are they necessarily congruent? No. Consider the following as a counter-example:



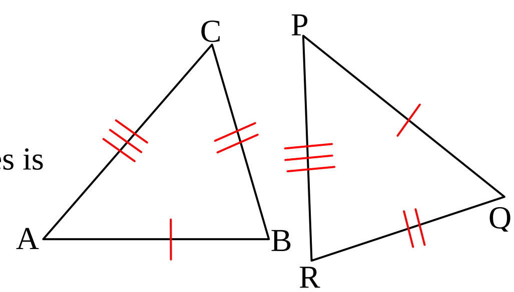
It is obvious that congruent corresponding angles are not sufficient. What about congruent corresponding sides? That, as they say, is a different story…

Postulate 4-1 Side-Side-Side (SSS) Postulate

If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.

$$\Delta ABC \cong \Delta PQR$$

So, yes having congruent corresponding sides is sufficient for congruent triangles.



A special word – "included"

Before we go any further, we need to be clear on how the term "included" is used. If we ask "which sides include $\angle B$" we are asking which sides $\angle B$ is <u>contained between</u>. In this sense, "included" implies location.

Example – not in book

In ΔVGB, which sides include $\angle B$?

> If it helps, draw the triangle out. However, this problem can be solved without drawing the triangle; in fact some problems may ask you to answer without drawing the figure! Angle B will be the vertex the two sides meet at.
>
> $\overline{GB}$ and $\overline{BV}$

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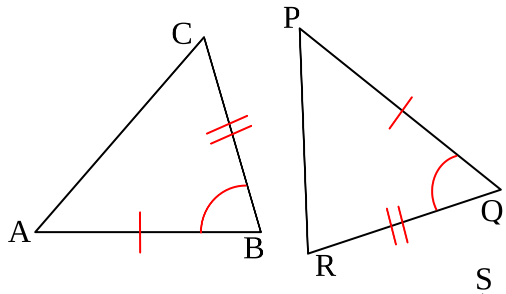
Another one: in ΔSTN, which $\angle$ is included between sides $\overline{NS}$ and $\overline{TN}$?

> This is pretty easy…just look at the two segments listed. They are two sides of the triangle. They share an endpoint, in this case N. Hence $\angle N$ is included between those two sides.

Postulate 4-2 Side-Angle-Side (SAS) Postulate

If two corresponding sides and the included angle of one triangle are congruent to the two corresponding sides and the included angle of another triangle, then the two triangles are congruent.

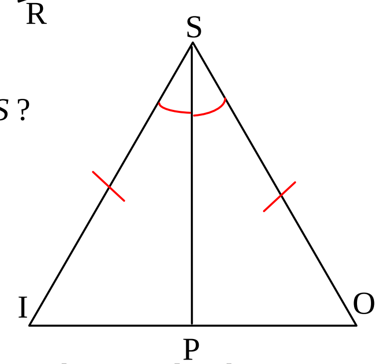
$$\Delta ABC \cong \Delta PQR$$



Here we see another way of determining that two triangles are congruent.

Remember, naming is critical!

As a case in point, consider the following example. Is $\Delta ISP \cong \Delta POS$?



Well, what have we got? We can assert the following:

$$\overline{IS} \cong \overline{OS},\ \angle ISP \cong \angle OSP \text{ and } \overline{PS} \cong \overline{PS} \text{ (reflexive POC)}$$

That's side-angle-side right? Yes, ***<u>but</u>*** we have another problem. This side-angle-side relationship is for specific corresponding sides. In the triangles the problem is asking about, are $\overline{IS}$ and $\overline{OS}$ corresponding sides? **NO THEY ARE NOT!** In ΔISP and ΔPOS, $\overline{IS}$ and $\overline{PO}$ corresponding sides!

What's the moral of the story? Make sure you are looking at corresponding sides.

Marking congruent parts

It is a very good idea to mark congruent parts as you are working through the diagram. If you find a side shared by both triangles (as $\overline{PS}$ is in the diagram above using the reflexive property of congruence) you can mark it with an X.

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Examples – not in the book

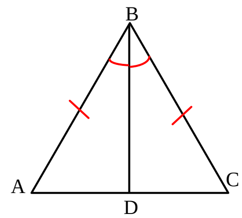
Decide whether you can use the SSS or SAS postulate to prove the triangles congruent.
If so, write the congruence statement and identify the postulate. If not, write *not possible*.

1. $\Delta ABD \cong \Delta CBD$, SAS

   $\overline{AB} \cong \overline{CB}$

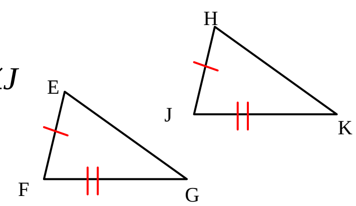
   $\angle ABD \cong \angle CBD$

   $\overline{BD} \cong \overline{BD}$



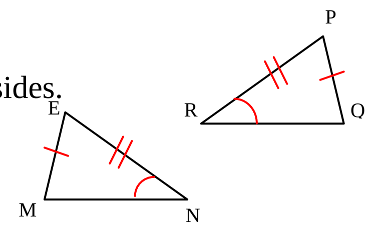
2. Not possible:

   $\angle F$ not $\cong \angle J$



3. Not possible:

   $\cong \angle$ not included between the congruent sides.

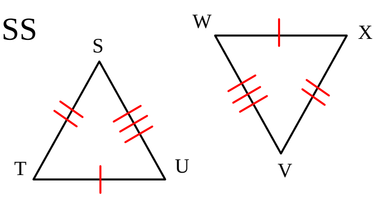


4. $\Delta STU \cong \Delta VXW$, SSS

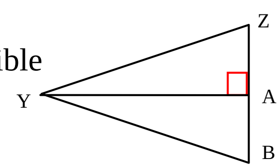
   $\overline{ST} \cong \overline{VX}$

   $\overline{TU} \cong \overline{XW}$

   $\overline{US} \cong \overline{WV}$



5. Not possible

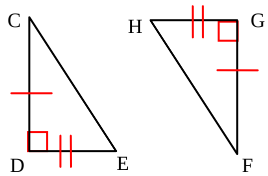


6. $\Delta CDE \cong \Delta FGH$, SAS

   $\overline{CD} \cong \overline{FG}$

   $\angle D \cong \angle G$

   $\overline{DE} \cong \overline{GH}$



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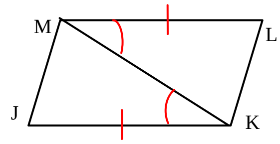
7. $\Delta JKM \cong \Delta LMK$, SAS

$\overline{JK} \cong \overline{LM}$

$\angle JKM \cong \angle LMK$

$\overline{MK} \cong \overline{MK}$ (Reflexive POC)

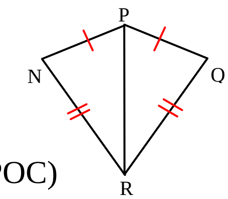


8. $\Delta PNR \cong \Delta PQR$, SSS

$\overline{PN} \cong \overline{PQ}$

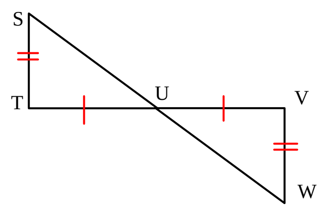
$\overline{NR} \cong \overline{QR}$

$\overline{RP} \cong \overline{RP}$ (Reflexive POC)



9. Not possible

No $\cong \angle' s$



Assign homework

p. 189 #1-4, 7-30, 33, 36, 38, 41, 44-47